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Hardness, Toughness, and Brittleness: An Indentation Analysis

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The ratio H/K_c, where H is hardness (resistance to deformation) and K_c is toughness (resistance to fracture), is proposed as an index of brittleness. Indentation mechanics provides the scientific basis for this proposal. The analysis, developed in terms of a model contact system, indicates that all materials are more susceptible to deformation in small-scale loading events and to fracture in large-scale events. By normalizing the characteristic dimensions of the two competing processes and the contact load in terms of appropriate functions of H and K_c, a universal deformation/fracture diagram can be constructed. From this diagram the mechanical response of any material of known hardness and toughness may be predicted for any prospective in-service contact loading conditions. The concept offers a simple approach to materials classification for design purposes.

I. Introduction

The advent of modern high technology has caused an upsurge in the development of strong materials. In an increasing number of engineering applications stringent service conditions, e.g. high temperatures and hostile environments, have necessitated a trend toward materials with intrinsically strong, covalent/ionic bonding. However, as structural components these materials suffer from one major disadvantage; they tend to be "brittle." Engineering design then becomes a question of containing the initiation and propagation of cracks.

Our knowledge of the factors which determine brittleness is largely empirical. Broadly, brittleness measures the relative susceptibility of a material to two competing mechanical responses, deformation and fracture; the abrupt "ductile-brittle transition" which occurs in many structural metals with decreasing temperature is a classic manifestation of this competition. However, although useful material parameters have been devised for quantifying both deformation and fracture properties as separate entities, surprisingly little effort has been made to combine such parameters into a common description. The crack-tip models of Kelly *et al.*[1] and Rice and Thomson,[2] which consider the local response of the structure about an equilibrium crack, represent the most serious attempts to resolve the issue of ductility vs brittleness at a fundamental level. In terms of these models, brittleness is tied up with the relative values of the ideal, reversible surface energy and the actual work of fracture. However, this concept does not lend itself to simple experimental analysis and is of more academic than practical importance.

An altogether different approach to the problem makes use of indentation theory. The basic idea, foreshadowed by several earlier workers (e.g. Douglas[3] and Peter[4]), stems from the realization that a suitable indentation pattern can provide simultaneous information on deformation and fracture properties of a given material. Two recent papers developed this idea analytically in accordance with Griffith-Irwin fracture mechanics: based on ideal "sharp-indenter" geometry, a formulation was given in terms of standard material parameters of the competing processes. In the first paper[5] both the deformation and fracture zones about the indenter contact area at any given load are taken as "well-developed": the scale of the residual impression is determined by the hardness, H, which quantifies the resistance to deformation; the scale of the cracking is determined by the toughness, K_c, which quantifies the resistance to fracture. The second paper[6] deals with the threshold conditions for fracture within the deformation-controlled indentation field: the onset of cracking occurs at a critical deformation zone size and is determined by both H and K_c. A size effect is thereby evident in the total indentation response, with the deformation process dominant at low indenter loads and the fracture process at high loads.

In the present study the separate propagation and initiation studies are combined into a unifying description and the ratio of hardness to toughness, H/K_c, is proposed as a simple index of brittleness. The main aim is to establish a convenient basis for materials classification, rather than to provide physical insight into the actual mechanisms of deformation and fracture involved.

II. Deformation/Fracture Parameters

Hardness and toughness are cited as convenient material parameters for characterizing deformation and fracture processes in a wide range of solids, from metals to ceramics. This choice of parameters may be justified in terms of the stress fields which drive the two processes (Fig. 1).

(1) Hardness as a Deformation Parameter

The routine hardness test is perhaps the simplest means by which irreversible deformation can be produced and measured in a controlled manner in any given material. The test uses a hard, sharp

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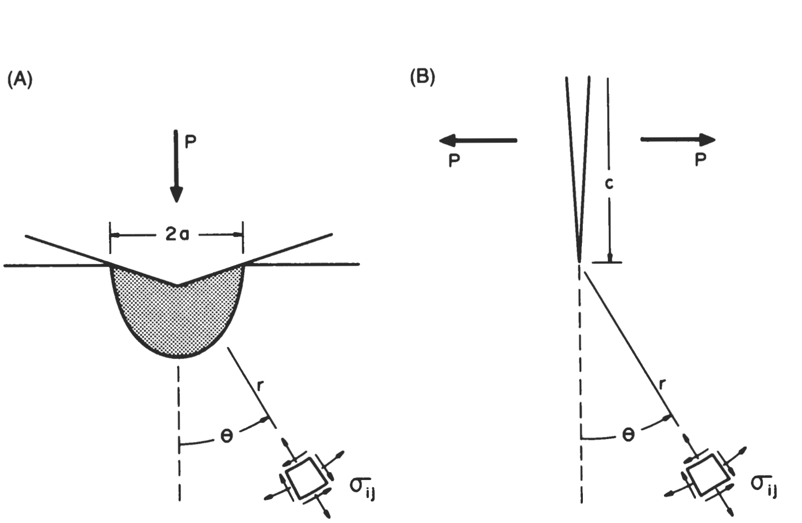

Fig. 1. (*A*) Indentation and (*B*) crack-tip fields. Dimensions a and c respectively characterize the scales of the deformation and fracture processes, and P characterizes the applied loading.

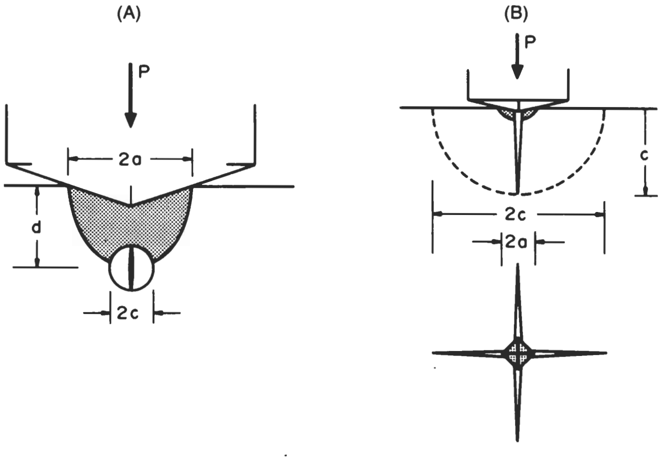

Fig. 2. Idealized deformation/fracture pattern for Vickers pyramid indentation, showing (A) initiation and (B) propagation stages of median crack development.

indenter and loads the specimen surface at a preselected position with a prescribed normal force. If the indentation were perfectly elastic the stress field would be singular at the loading point; the system avoids this by deforming beneath a nonzero contact area, thereby leaving a residual impression. With indenters of regular geometry the mean contact pressure is load-independent ("geometrical similarity")[7] and accordingly gives a measure of the material hardness:

$$H = P/\alpha_0 a^2 = \text{constant} \tag{1}$$

where P is the load, a a characteristic dimension of the impression (Fig. 1(A)), and α_0 an indenter constant. The stress field about the contact site then has polar components of the form[8]

$$\sigma_{ij} = H f_{ij}(\theta, a/r) \tag{2}$$

where the f_{ij} are dimensionless coordinate functions. Since it uniquely determines the intensity of the field, H is a useful indicator of the driving force for the deformation process.

From a physical viewpoint, the hardness test may be seen as responding to the dominant mechanism of deformation in any given material. In ideally plastic solids hardness relates directly to the yield stress,[7] the quantity traditionally used by engineers and metallurgists to specify the resistance to slip processes. With some of the more brittle ceramics, however, this correspondence is in doubt. In conventional stress/strain tests ceramic specimens tend to fracture before yield can occur: in the extreme case of ideally covalent solids there is evidence that flow processes cannot operate at all below the limits of theoretical shear stress at room temperature. When elaborate precautions are taken to suppress all components of tensile stress (e.g. by conducting tests under confining pressures) some solids, notably those with "open" structures, tend instead to deform by structural densification. Since the indentation stress field contains comparable components of shear and hydrostatic compression,[8] it is not always readily apparent which mode of deformation is responsible for the residual impression: in silica glass, for instance, the nature of the deformation zone has become an issue of considerable controversy.[9,10] Notwithstanding these interpretive complications, hardness is the most accessible of all deformation parameters in the general classification of strong materials.

(2) Toughness as a Fracture Parameter

With the development of Griffith-Irwin fracture mechanics, several parameters have become available for specifying resistance to crack growth. Of these parameters, K_c has gained the widest acceptance as a material quantity in design. Toughness identifies with the stress intensity factor for an equilibrium tensile crack and thereby takes on the form[11]

$$K_c = P\xi(c) = \text{constant} \tag{3}$$

where P is a loading parameter and c is a characteristic crack dimension (Fig. 1(B)): the function ξ depends on test-piece geometry. In analogy to H in Eq. (1), K_c uniquely determines the intensity of the stress field about the crack tip,

$$\sigma_{ij} = K_c g_{ij}(\theta)/(2\pi r)^{1/2} \tag{4}$$

where the g_{ij} are angular functions; thus K_c represents the fracture driving force.

In terms of fracture mechanics, the toughness parameter may be interpreted physically at two levels[11]: macroscopically, it emerges as a composite of elastic and surface-formation properties; microscopically, it relates to the basic nonlinear separation processes responsible for crack-tip extension. For a solid containing a well-defined crack of specified dimensions, K_c determines the fracture stress in uniform tensile loading and is accordingly a key material quantity in strength analysis. Although originally derived in a historical context of structural metals,[12] the potential usefulness of toughness as a general fracture parameter has been amply demonstrated by the more recent applications in ceramic engineering.[13]

III. Indentation Mechanics

Given H as a measure of resistance to deformation and K_c of resistance to fracture, it is reasonable to ask whether comparative values of these two parameters might be taken as an indicator of brittleness. One difficulty is immediately apparent: H and K_c have different dimensions, such that the ratio H/K_c has dimensions of (distance)$^{-1/2}$. However, a definite physical significance may be attached to this (distance) factor in the indentation problem[5,6,14,15]: H/K_c reflects on the relative scales of the deformation and fracture zones about a sharp-contact site and thereby introduces a size effect into the competitive mechanical responses. The present aim is to develop this scaling concept into a general formulation.

(1) Universal Deformation/Fracture Diagram

Consider the evolution of the deformation/fracture pattern for a sharp-indenter system (Fig. 2). The specific choice of the Vickers pyramid geometry in the present work allows for standardization of results. Empirically, the Vickers system maintains geometrical similarity in the deformation zone, even after the onset of cracking, provided the indented material is reasonably homogeneous over the scale of the impression.

The crack pattern develops in two distinct stages during indenter loading, viz. initiation (Fig. 2(A)) and propagation (Fig. 2(B)). The indentation stress field, although dominated by the shear and hydrostatic components, necessarily contains a small amount of tension, so the potential for fracture exists in any contact event.[8] With the Vickers indenter this tension concentrates on median planes containing the diagonals of the square-shaped surface impression, producing the characteristic radial star pattern shown schematically in the surface view of Fig. 2(B). Direct observations of the indentation process in glass show that these so-called "median" cracks initiate from subsurface nucleation centers below the contacting point. The centers themselves are generally created by the actual deformation processes responsible for the hardness impression,[16] although fortuitous preexisting flaws can sometimes act as suitable nuclei.[15] Ideally, the initiation cracks start as full pennies at the boundary between deformation zone and surrounding elastic matrix (Fig. 2(A)), where the tensile stresses are maximum. At a critical zone size the subsurface pennies become unstable and subsequently expand toward the indented surface. Ultimately, the stable, fully propagating, half-penny configuration of Fig. 2(B) is attained. There are some variants on this idealized geometrical pattern; e.g. shallow "radial" cracks may grow outward from the impression corners without propagating fully downward (especially on "rough" surfaces),[15] or "lateral" cracks, driven by a residual "mismatch" stress field about the deformation zone, may initiate and grow toward the specimen surface as the indenter is being unloaded.[8] However, these variants are not expected to have a serious modifying influence on the half-penny configuration which forms the basis of our model system.

Detailed analyses of the deformation/fracture mechanics of the indentation process shown in Fig. 2 provide equilibrium relations

Table I. Hardness, Toughness, and Brittleness Parameters*

		Deformation/fracture parameters			Threshold parameters	
Material	Comments	H (GPa)	K_c (MPa m$^{1/2}$)	H/K_c (μm$^{-1/2}$)	P^* (N)	c^* (μm)
Fe	Medium-strength steel	5	50	0.1	800 000	12 000
NaCl	Monocrystal	0.24	0.4	0.6	30	330
ZnSe	Vapor-deposited	1.1	0.9	1.2	8	80
WC	Co-bonded	19	13	1.4	70	60
ZnS	Vapor-deposited	1.9	1.0	2	2	30
Si_3N_4	Hot-pressed	16	5	3	2	12
Al_2O_3	MgO-doped	12	4	3	2	12
SiC	Hot-pressed	19	4	5	0.6	5
MgF_2	Hot-pressed	5.8	0.9	6	0.05	3
MgO	Hot-pressed	9.2	1.2	8	0.04	2
SiO_2	Glass	6.2	0.7	9	0.02	1.5
B_4C	Hot-pressed	77	6	13	0.05	0.7
Si	Monocrystal	10	0.6	17	0.002	0.4

*Data courtesy A. G. Evans, University of California, Berkeley.

for the characteristic dimensions a and c in terms of load P. The appropriate relation for the deformation zone is (from Eq. (1)):

$$P/a^2 = \alpha_0 H \tag{5}$$

For the fracture the appropriate form of Eq. (3) must be derived separately for the initiation[6] and propagation[17] stages. Taking the propagation stage first, the well-developed median cracks extend under near center-loading conditions and accordingly satisfy the simple equation for pennylike geometry

$$P/c^{3/2} = \beta_0 K_c \qquad (c >> a) \tag{6}$$

where β_0 is another indenter constant. The $c(P)$ function appropriate to initiation is much more complex: its key feature is a minimum in the load at

$$P^* = \lambda_0 K_c (K_c/H)^3 \tag{7a}$$

$$c^* = \mu_0 (K_c/H)^2 \tag{7b}$$

with λ_0 and μ_0 as additional geometrical constants. Now, by suitably normalizing the indentation variables in Eqs. (5) to (7), load in terms of $K_c(K_c/H)^3$ and linear dimensions in terms of $(K_c/H)^2$, the computed functions can all be represented on a single, universal diagram. Since not all of the indentation constants in these equations can be calculated accurately, adjustments are made in accordance with available Vickers data from some of the materials listed in Table I. That is, $\alpha_0 = 2$ exactly, taking a as the half-diagonal of a square impression and defining H in Eq. (1) in terms of load over projected area of contact (in accordance with Tabor's assertion that the true mean contact pressure is the more fundamental measure of hardness[7]—the more conventional VHN, based on the actual impression area, gives a value some 6% lower); $\beta_0 = 7$, best fit to data, accurate to within a factor of two or three (cf. $\beta_0 = 19$ for an ideal half-penny crack with Vickers loading, calculated ignoring effects due to contact friction, presence of free surface, residual stresses about deformation zone, etc.[17]); $\lambda_0 = 1.6 \times 10^4$ and $\mu_0 = 120$, best fit to data with constraint $c^* \approx a$ (in line with the mechanism of deformation-induced crack nucleation), accurate to within about an order of magnitude (cf. $\lambda_0 = 2.2 \times 10^4$ and $\mu_0 = 44$ for approximate calculation based on ideal penny median crack nuclei[6]). The results are shown in Fig. 3. Threshold parameters evaluated from Eq. (7) on the basis of these adjustments are included in Table I.

(2) *Size Effect and Its Implications in Design*

The universal plot of Fig. 3 is convenient for assessing the relative susceptibilities to deformation and fracture of any given material. Once the parameters H and K_c are specified, the scale of the damage process fixes the position on the diagram and hence determines the mechanical response. For the purpose of design against any contact-related property the threshold parameter P^* must be reckoned in terms of the characteristic service load P; three distinct regions of mechanical behavior may accordingly be identified on the diagram.

(A) $P^* > P$ (Small-Scale Events): The spatial scale of the contact is insufficient to extend crack nuclei,[6] and the damage is therefore entirely deformation-controlled. Optimal design then calls for maximization of H in Eq. (5). Most metallic (and polymeric) solids tend to fall into this category.

(B) $P^* << P$ (Large-Scale Events): Median cracks are well developed, with $c >> a$, so the damage is essentially fracture-controlled. One then seeks to maximize K_c in Eq. (6). This domain is that of the typical covalent solid.

(C) $P^* \approx P$ (Intermediate-Scale Events): This region is the threshold, where fracture and deformation operate on a comparable scale. In this case it is the composite quantity K_c/H in Eq. (7) (and, to a lesser extent, K_c in Eq. (7a)) which should be maximized.

It is in the context of these considerations that the dimension c^* in Fig. 3 becomes a parameter of special significance. When the size of

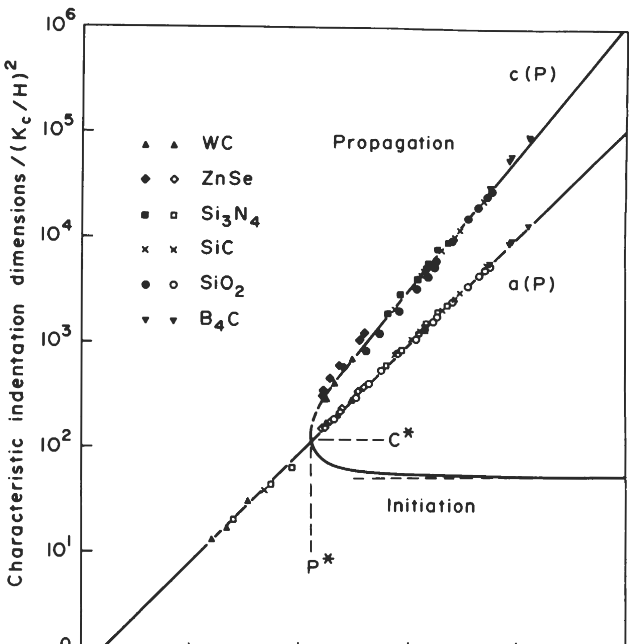

Fig. 3. Universal deformation/fracture diagram. Deformation curve $a(P)$ (open symbols) is according to Eq. (5), fracture curve $c(P)$ (closed symbols) according to Eq. (6) in propagation region and to appropriate function from Ref. 6 in initiation region (broken curve is smoothing function representing ill-defined transition region between Figs. 2(*A*) and 2(*B*)). Data for crystalline solids courtesy A. G. Evans, University of California at Berkeley, that for glass (soda-lime) courtesy A. Arora, University of New South Wales.

crack nuclei within the deformation zone, as determined by the dimension a, becomes comparable with c^* the mechanical response is on the verge of transforming from hardness-controlled to toughness-controlled. The ratio K_c/H in Eq. (7*b*) is accordingly seen as the governing factor in determining the question of ductility vs brittleness in the contact problem. The inverse quantity, H/K_c, then becomes a convenient "index of brittleness"; thus in Table I the materials are listed in order of increasing brittleness.

IV. Discussion

Using indentation mechanics, a simple index of brittleness has been derived in terms of basic hardness and toughness parameters. Implicit in the formulation is an underlying geometrical similarity in the deformation[7] and fracture[6,17] processes, i.e. meaningful comparisons between materials require invariance in the indentation pattern. In practice, departures from the idealized geometry of Fig. 2 are not uncommon, especially in tests on anisotropic, inhomogeneous materials (e.g. monocrystals, coarse-grained polycrystals, or second-phase materials).[15] Kinetic effects can add to the complication by effectively reducing the hardness and toughness below their equilibrium values, thus enhancing the scale of the indentation damage at any given load.[18] However, even allowing for a conservative uncertainty factor of two in estimates of H/K_c, distinctions can be made between most materials (other than those close together) in Table I.

A notable feature of this brittleness concept is its amenability to routine experimentation, with all the advantages of simplicity and economy characteristic of indentation testing. Indeed, all the information needed for a determination of the brittleness index is contained in a single, well-developed Vickers indentation pattern, as may be seen by combining Eqs. (5) and (6):

$$H/K_c = (\beta_0/\alpha_0 a^{1/2})(c/a)^{3/2} \qquad (c >> a) \tag{8}$$

In principle, therefore, H/K_c may be evaluated from scale measurements alone, without even specifying the load. However, in view of the acknowledged uncertainties associated with the indentation mechanics, any such evaluation must be regarded as no more than a first approximation.

Although defined here explicitly in terms of the contact problem, the index H/K_c appears to have a certain generality in the formulation of the ductility vs brittleness question. It can readily be inferred from Section II that this quantity must relate directly to the relative values of the yield (or compaction) and fracture stresses for any solid of specified crack geometry: the size of the dominant flaw then enters as a critical scaling factor in the normalization of the index. In another description, based on Marsh's presumed equivalence between crack-tip and sharp-contact deformation micromechanics,[9] a critical nonlinear zone size $\propto (K_c/H)^2$ determines the condition for crack growth (however, this approach presupposes an ideally plastic response in the corresponding deformation characteristics, and is currently the subject of some controversy, particularly in relation to ceramic solids).[19] The inevitable appearance of a critical linear dimension in each formulation highlights the fact that deformation is volume-controlled, whereas fracture is surface-controlled, i.e. in a geometrically similar situation the scale of the competing processes will vary with mechanical energy input to the one-third and one-half powers respectively, making fracture the more rapidly varying of the two.

In the interpretation of P^* as the maximum load and c^* as the associated maximum flaw size that a solid might sustain in a contact event without the onset of fracture (Section III), the potential role of hardness and toughness data in the selection of materials for design purposes was indicated. As a first step, some estimate must be obtained of a characteristic contact load or dimension under prospective service conditions. Bearing in mind that the design specifications may include other than purely mechanical requirements, one may then identify a region of Table I from which a final choice of material may be made. Thus, if in any particular application it is necessary to avoid contact-induced cracking at all costs, e.g. to minimize strength loss, surface erosion, and wear, a material of high hardness should be sought toward the top of the table. Conversely, if a particular specification such as optical transparency, electrical resistance, or high melting point were to be the overriding design factor, one might be limited to a choice from the brittle end of the spectrum, in which case high toughness becomes the goal. Of particular interest here are materials, such as ZnSe, which may be considered a poor candidate for structural components because of low resistance to both deformation and fracture; in any application where immunity to crack *initiation* is the prime consideration, as perhaps in the maintenance of optical integrity in ir windows, such a material may serve well because of its low brittleness index.

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